Pre-equilibrium evolution of non-Abelian plasma

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Abstract

We study the production and the equilibration of a non-Abelian $q\bar{q}$ plasma in an external chromoelectric field, by solving the Boltzmann equation with the non-Abelian features explicitly incorporated. We consider the gauge group $SU(2)$ and show that the colour degree of freedom has a major and dominant role in the dynamics of the system. It is seen that the assumption of the so called Abelian dominance is not justified. Finally, it is also shown that many of the features of microscopic studies of the system appear naturally in our studies as well.

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I. INTRODUCTION

There is a high expectation that with the advent of new machines (RHIC at BNL, LHC at CERN) it will be possible to study the quark gluon phase of strongly interacting matter. Lattice QCD predicts that at density and temperature of the order of $2GeV/fm^3$ and 200 MeV respectively, hadronic matter undergoes a transition to the quark-gluon phase [1]; the Au-Au collisions at 200 GeV/nucleon (in the centre of mass frame) in RHIC are expected to provide such energy densities which would then give rise to the quark- gluon phase [2]. It is also generally believed that the transition is of the first order, although more recent lattice studies [3] and those of intermittency [4] suggest that this belief needs more substantiation.

The most important question experimentally is of course the signature of quark gluon plasma (QGP). Many proposals have been made, prominent among them being J/Ψ suppression [5], strangeness enhancement [6] and dilepton production [7]. Clearly, it is important to know, even if only roughly, the dynamics of the QGP evolution in space - time. Indeed, apart from being intrinsically interesting, the space - time dynamics will have a direct impact on the signatures proposed.

The space time evolution of QGP that is produced in ultra-relativistic heavy ion (URHIC) collisions can be described [8] broadly in four stages. They are, i) pre-equilibrium, ii)equilibrium, iii) cooling, and iv) hadronisation. While most of the earlier work refers to stage (ii) where one studies the hydrodynamic evolution [9], subsequently there has been a good deal of effort in understanding stage (i) as well. Since it is of relevance to us here, we briefly reveiw them below.

In the context of Flux-tube model (described in the next section), the transport equation is solved to study the evolution of the system. Assuming the Bjorken scenario [10] where a central rapidity plateau will be seen in relativistic heavy ion collisions [11], this model deals with a baryon free plasma in the central region; there would initially be a huge deposit of energy which then creates the partons. In the approach of Baym [9], these partons are assumed to be created soon after the two nuclei have collided with each other, and are

further assumed to equlibrate almost instantaneously thereafter (with a very small relaxation time). This quark-gluon gas then proceeds to expand hydrodynamically. Subsequent work of Kajantie and Matsui [12], incorporated a dynamic particle production by introducing a source term via the Schwinger mechanism [13] in the Boltzmann equation, and they also studied the equilbration in the relaxation time approximation. On the other hand, Bialas *et al* [14] generalized the Baym analysis by introducing the effect of the background field on the otherwise hydrodynamic flow. A self consistent study of the system was carried out by Banerjee *et al* [15] who combined the effect of both the background field and the collisions on the quark-antiquark system. Note that in the analyses of Kajantie *et al* [12] and Banerjee *et al* [15], the source term acquires a time dependence by virtue of the time dependence that the electric field suffers because of the particle production. Finally, by employing the same analysis as of Banerjee *et al*, Asakawa and Matsui [16] have studied the rate of dilepton production by $q\bar{q}$ annnihilation, as a function of (proper) time. All the above analyses ignore the gluon component.

In contrast to the above studies which we shall broadly term macroscopic, there is an interesting and a more complete as well as a complementery approach by Geiger and Kapusta [17]. They have studied the QGP evolution in what they call the parton cascade model. In this approach, the production and the evolution of q$\bar{q}$ pairs and gluons are studied numerically, by considering the direct collision processes $2 \to 2$, the inelastic fusion processes $2 \to 1$, and the decay processes $1 \to 2$. The above processes are studied within the frame work of the perturbative QCD, by taking the leading order and the next to leading order contributions. While this approach of Geiger and Kapusta has the merit of being microscopic, it comes with the complication of extensive numerical computation - and the physical picture may not always be clear. More significantly for our purposes, the analysis confirms the Bjorken hypothesis of the boost invariance of the parton densities in the central rapidity region. Further, the relaxation time which is other-wise free in the flux-tube models also gets determined. Indeed, Geiger et al [17] report a value of $\sim 0.2 fm$ in their paper. One may thus look upon the flux tube models to be effective versions of the more detailed

microscopic pcm, if the τ_c determined by the latter is employed in the transport equation. We shall make a more detailed comparison in the subsequent sections.

In any case it is our purpose to emphasize and study a crucial feature of the quark-gluon phase, *viz*, the inherently non- Abelian nature of the plasma. Note that all the earlier macroscopic approaches [9,12,14–16] are in the so called Abelian approximation. Put simply, the non-Abelian features are completely ignored in the space time evolution, and the Boltzmann equation describes the dynamics which is essentially that of a Maxwell fluid, be it in the source term or in the background field term. The approach of Geiger and Kapusta [17] does incorporate the non-Abelian features in their basic Feynman diagrams, but is neverthless incomplete in that the distribution function in its final form does not carry any color degee of freedom.

We attempt to fill this gap, within the framework of transport equations, by explicitly incorporating the non-Abelian features mentioned above. To be sure, the relevant formalism has already been devloped by Heinz [18] . Here one needs to carefully implement the non-Abelian constraints within the Boltzmann equation such that the Bjorken picture of boost invariance [10] along the collision direction is also preserved.

Before we proceed to present the formalism it is pertinent to make the following observations: 1) Strictly speaking, the source term that has been employed in the above studies is inadmissiible. For, even within the so called Abelian approximation, the chromoelectric field suffers a space time evolution in any self-consistent calculation. To wit, the production of particles is then strictly governed by a time dependent field, in which case a perturbative mechanism will take over from the non-perturbative Schwinger mechanism. Note that the latter holds only for a uniform constant field. All the previous studies employing the flux tube model employ the non-perturbative expression. 2) As already mentioned, the source term is derived for QED processes and needs to be redone for QCD processes at hand. It is apparent that the QCD effect will show up most manifestly in the gluon production; there is no corresponding counterpart in QED. In other words it is necessary to obtain a source term for gluons. 3) It is well known from earlier studies of Yang-Mills equations that there

are inequivalent gauge field configurations which yield the same field tensor [19], and hence the same energy momentum tensor. This feature which is again absent in the Maxwell case has to be taken into account in our study. Recall that the lattice studies at any given energy density implicitly sum over the contributions coming from all such inequivalent configurations [20]. 4) Finally, the inherent non-linearity and the local gauge invariance which characterize the Yang-Mills equations lead to an added complexity; the equations do not possess a unique solution, even after gauge fixing, and even if one considers only static configurations. Consequently, one needs to scan, with some suitable weightage, all the solutions for the given initial conditions, and sum over all such configurations. It is not clear if any procedure is known that allows an implementation of this requirement.

In this first study we restrict ourselves to a study of the simplest of the situations leaving the points mentioned above for a later presentation. Apart from acting as a warm up, it will also allow us to devise a simple bench mark that would enable us to compare the Abelian and the non-Abelian features. With that in mind we consider only Maxwell like fields (see below), ignore the production due to a time dependent field and, finally restrict ourselves to the gauge group $SU(2)$. The last choice leads to an additional simplification as quarks, antiquarks and gluons all belong to the same represantation. As a final simplification, we shall also drop the gluonic terms altogether. Please note that even with all these approximations (which we shall remedy in later publication) our approach is none-the worse compared to the fully Abelian models which have been employed so far.

The plan of the paper is as follows. In the next section, we shall briefly describe the model where we set up the transport equation with the non-Abelian features appropriate to the group $SU(2)$ explictly incorporated. We also obtain the auxiliary equation that follows from the requirement of the conservation of the energy momentum tensor. Section **3** describes briefly the numerical procedure employed. In Section 4 we present the results and discussions. We conclude and summarize the main results in Section 5.

II. THE FLUX TUBE MODEL

One well known signature for the QGP is the dilepton production [7] which has the attractive feature that the leptons do not suffer any final state (strong) interaction. As such this process can therefore be used to study the space-time evolution of the QGP. In particular, one would be able to study the pre-equilibrium regime, provided one can determine the distribution functions for the quarks and the anti-quarks at such early times.

The flux tube model, as we have mentioned, is a useful choice in this context. We shall employ this model in this paper. A brief description of the Flux Tube model is not out of place here.

In this model the two nuclei that undergo a central collision at ultra high energies are lorentz contracted as thin plates [21,22]. These two lorentz contracted nuclei pass through each other and, in the process, acquire a non-zero colour charge ($\langle Q \rangle = 0$, $\langle Q^2 \rangle \neq 0$), by exchanging soft gluons. So one may figuratively call such nuclei after collision as color capacitor plates between which a strong chromoelectric field is created. This field creates q$\bar{q}$ and gluon pairs *via* the Schwinger mechanism which enforces the instability of the vacuum in the presence of an external electric field. The partons so produced will collide with each other and also get accelerated by the parent background field. The mutual collisions drive the system towards equilibrium with suitable modulation from the background acceleration.

Let $f(\bar{f})$ be the one particle(anti-particle) distribution in the phase space. In the earlier Abelian approximation, the evolution of the particles is studied via the Boltzmann-Vlasow equation,

$$\left[p_\mu \partial^\mu \pm g F_{\mu\nu} p^\nu \partial_p^\mu\right] f(x,p)(\bar{f}(x,p)) = C(x,p) + S(x,p), \tag{1}$$

where g is the strength of the charge which couples to the field. Note that the distribution functions f, $\bar{f}$ are defined in a six dimensional phase space of coordinate and momenta. In the above equations, C is the collision term driving the system into equilibrium, and S is the term that acts as the source for pair production [13]. We now generalize Eqn. (1) to

incorporate the non-Abelian features explicitly. An easy way to to accomplish this is to extend the phase-space by taking the color degrees of freedom into account. The extended phase space is now a direct sum $\mathcal{R}^6 \oplus \mathcal{G}$, where $\mathcal{G}$ is the (compact) space corresponding to the given gauge group. Thus the single particle phase space has a dimension $d = 6+(N^2-1)$ if we consider $SU(N)$. The evolution of the color charge now follows Wong's equation [23]

$$\frac{dQ^a}{d\tau} = f^{abc} u_\mu Q^b A^{c\mu} \tag{2}$$

which supplements the Lorenz force equation

$$\frac{dp^\mu}{d\tau} = Q^a F^{a\mu\nu} u_\nu \tag{3}$$

where u_μ is the four-velocity and f^{abc} are the structure constants of the gauge group. For SU(2) charges that we are interested in, the non-Abelian extension of Eqn.(1) reads

$$\left[p_\mu \partial^\mu + Q^a F^a_{\mu\nu} p^\nu \partial^\mu_p + \epsilon^{abc} Q^a A^b_\mu p^\mu \partial^c_Q\right] f(x,p,Q) = C(x,p,Q) + S(x,p,Q) \tag{4}$$

Note that we do not have to write a separate equation for anti-quarks since they belong to the same color representation as quarks. Indeed, the antipodal points on the sphere (corresponding to the color part of the phase-space with a fixed charge) represent the particle and anti-particle. Eqn. (3) may be recast in the more convenient form

$$\frac{d\vec{Q}}{d\tau} = u_\mu \vec{Q} \times \vec{A}^\mu \tag{5}$$

where the arrows now denote the direction in the color space. In the same notation,the transport equation obtains the form

$$\left[p^\mu \partial_\mu + \vec{Q} \cdot \vec{F}_{\mu\nu} p^\mu \partial^\nu_p + p^\mu \vec{Q} \times \vec{A}_\mu \cdot \frac{\partial}{\partial \vec{Q}}\right] f(x,p,\vec{Q}) = C(x,p,\vec{Q}) + S(x,p,\vec{Q}) \tag{6}$$

For the model at hand, let us take the 'plates' to be moving along the z-direction. As mentioned earlier, we restrict $\vec{F}^{\mu\nu}$ to be Maxwell like, *i.e.*, we restrict to only that field configuration for which there exists a gauge choice such that the gauge potentials commute with each other every where. We next require a boost invariant description of the distribution

functions as well as the other physical quantities which may be determined thereof, even as the system is evolving. Keeping this in mind, we make the gauge choice where only the components $A^{\mu a} = (A^{03}, A^{33})$ are non-vanishing and the other components are zero. With this choice, the resulting electric field points in the '3' direction in the colour space. We now proceed to impose the Lorentz gauge condition, which has to be done such that the chromoelectric field depends on the boost-invariant(along the axis of collision) quantity $\tau = (t^2 - z^2)^{1/2}$. Observing that the system is effectively (1+1) dimensional, in the t-z plane, we write

$$\vec{A}^\mu = \epsilon^{\mu\nu}\partial_\nu \vec{G}(\tau) \tag{7}$$

where the indices μ, ν take values 0, 3 and $\vec{G}$ is a Lorentz scalar function depending only on τ. The above choice automatically implements the Lorentz gauge condition $\partial_\mu \vec{A}^\mu = 0$. Clearly, the chromoelectric field $\vec{E}$ is dependent only on τ, and is given by

$$\vec{E}(\tau) = \left[\frac{d^2}{d\tau^2} - (2/\tau)\frac{d}{d\tau}\right]\vec{G}(\tau) \tag{8}$$

The Wong equation guarantees the conservation of the magnitude of the vector charge $\vec{Q}$, which may now be held fixed. Being the analogue of the Larmor equation for a charged particle in an external magnetic field, it also conserves the component of the charge that is parallel to the external chromoelectric field. It is therefore convenient to resolve the SU(2) charge in the polar coordinates. Writing

$$Q_1 = Qsin\theta cos\phi;\; Q_2 = Qsin\theta sin\phi;\; Q_3 = Qcos\theta\,, \tag{9}$$

it is straight forward to verify that

$$\left(\vec{Q} \times \frac{\partial}{\partial \vec{Q}}\right)_3 = \frac{\partial}{\partial \phi} \tag{10}$$

The last equation will lead to considerable simpification in solving the transport equation.

Let us now consider the collision term. As mentioned in the introduction, the colour plate model can be looked upon as an effective version of the more detailed parton cascade

model which has confirmed the Bjorken hypothesis. We may therefore, employ the simple relaxation time hypothesis, with the phenomenological parameter τ_c, the relaxation time, to be obtained from microscopic computations. It then follows that

$$C = \frac{-p^{\mu}u_{\mu}(f - f_{eq})}{\tau_c} \tag{11}$$

where f_{eq} is the eqilibrium distribution function, with local (space time dependent)values of the thermodynamic quantities. While it has been customary to take f_{eq} to be one of an ideal gas, with some support from the pcm analysis of Geiger and Kapusta [17], recent lattice studies suggest that the quark-gluon phase is possibly a non-perturbative phase. Since we are not dealing with a true system here, we shall conveniently take f_{eq} to be that of an ideal Fermi gas with a local temperature, evolving as a function of the proper time τ. We thus have

$$f_{eq} = \frac{2}{\exp(p^{\mu}u_{\mu}/T(\tau)) + 1}, \tag{12}$$

where u^{μ} is the flow velocity,

$$u^{\mu} = (cosh\eta, 0, 0, sinh\eta). \tag{13}$$

which is written in terms of the (space-time) rapidity $tanh\eta = z/t$.

Now we demand the boost invariance following Bjorken's picture according to which the longitudinal boosts are the symmetry operations on the single particle distribution. The boost invariant parameters on which f can depend are, apart from the charge coordinates,

$$\tau = (t^2 - z^2)^{1/2},\, \xi = (\eta - y),\, p_t = (p_0^2 - p_l^2)^{1/2} \tag{14}$$

where $y = \tanh^{-1}(p_l/p_0)$ is the momentum rapidity. It is convenient to write separate equations for quarks and anti-quarks, *a la* the Abelian case. If we therefore identify the quark states with the points on the upper hemisphere of the colour sphere, and the antiquarks with the lower hemisphere, by a trivial relabeling, we may write two equations,

$$\left[\frac{\partial}{\partial\tau} - \left(\frac{\tanh\xi}{\tau} + \frac{g\cos\theta E(\tau)}{p_t\cosh\xi}\right)\frac{\partial}{\partial\xi} + g\frac{d}{d\tau}G(\tau)\tanh\xi\frac{\partial}{\partial\phi}\right] f(\tau,\xi,p_t,\theta,\phi) \tag{15}$$

$$+\frac{f}{\tau_c} = \frac{f_{eq}}{\tau_c} + \frac{\Sigma(\tau,p_t,\xi,\theta)}{p_t\cosh\xi} \tag{16}$$

$$\left[\frac{\partial}{\partial\tau} - \left(\frac{\tanh\xi}{\tau} - \frac{g\cos\theta E(\tau)}{p_t\cosh\xi}\right)\frac{\partial}{\partial\xi} + g\frac{d}{d\tau}G(\tau)\tanh\xi\frac{\partial}{\partial\phi}\right] \bar{f}(\tau,\xi,p_t,\theta,\phi) \tag{17}$$

$$+\frac{\bar{f}}{\tau_c} = \frac{f_{eq}}{\tau_c} + \frac{\Sigma(\tau,p_t,\xi,\theta)}{p_t\cosh\xi} \tag{18}$$

with the first of them for the quarks and the next for the anti-quarks. The angle variable θ varies from 0 to $\pi/2$ in both the equations. Finally, Σ is the (non perturbative) Schwinger's expression for pair production and is given by

$$\Sigma(\tau,\xi,p_t,\theta) = -\frac{gE\cos\theta}{8\pi^3}\ln\left[1-\exp\left(-\frac{2\pi p_t^2}{gE\cos\theta}\right)\right](\frac{\alpha}{\pi})^{1/2}\exp(-\alpha\xi^2) \tag{19}$$

where we have inserted the Gaussian dependence on ξ by hand. Note that the non-Abelian nature features dominantly *via* the $cos(\theta)$ term. This occurence plays an important role in the evolution of the system.

After having set up the relevant equations (not all yet since energy-momentum conservation is to be imposed), we observe that the above differential equation possesses the formal solution

$$f(\tau,\xi,p_t\theta,\phi) = \int_0^\tau d\tau'\exp(\frac{\tau'-\tau}{\tau_c})\left[\frac{\Sigma(\tau',\xi',p_t,\theta)}{p_t\cosh\xi'} + \frac{f_{eq}(\tau',\xi',p_t)}{\tau_c}\right] \tag{20}$$

where $\xi(\tau\prime)$ is given by

$$\xi' = \sinh^{-1}\left[\frac{\tau}{\tau'}\sinh\xi + \frac{g\cos\theta}{p_t\tau'}\int_{\tau'}^{\tau} d\tau'' E(\tau'')\right] \tag{21}$$

It is thus clear that f does not depend on ϕ, so the $\frac{\partial}{\partial\phi}$ term contributes nothing to the transport equation. However the θ dependence is still involved through out the formalism which shows the non-Abelian effects. The corresponding distribution $\bar{f}$ for antiquark can be written by just changing g to $-g$ in f as can be checked out easily.

The transport equation has to be supplemented with the conservation constraint

$$\partial_\mu T^{\mu\nu}_{mat} + \partial_\mu T^{\mu\nu}_{YM} = 0, \tag{22}$$

since it is the field energy that is being pumped in order to produce the $q\bar{q}$ pairs. More explicitly,

$$\partial_\mu T^{\mu\nu}_{mat} = -j^a_\mu F^{\mu\nu}_a \equiv -\partial_\mu T^{\mu\nu}_{YM} \tag{23}$$

with

$$T^{\mu\nu}_{mat} = \int p^\mu p^\nu (f + \bar{f}) d\Gamma d\Omega_Q \tag{24}$$

where the measures $d\Omega_Q = \sin\theta d\theta d\phi$, $d\Gamma = \frac{\gamma d^3p}{(2\pi)^3 p_0} = \frac{\gamma p_t dp_t d\xi}{(2\pi)^2}$ and $j^\mu_a = \int p^\mu Q_a (f - \bar{f}) d\Gamma d\Omega_Q$. Here we have taken the value of the degeneracy factor $\gamma = 2$ corresponding to two flavours. Since energy and momentum are conserved in each collision, the moment of the sum of the collision term vanishes:

$$\int p^\nu (C) d\Gamma d\Omega_Q = 0 \tag{25}$$

Now taking the first moments of the Boltzmann equation and integrating over the color degrees of freedom for f and $\bar{f}$ and making use of the conservation of energy and momentum, we obtain

$$\partial_\mu T^{\mu\nu}_f + gE(\tau) \int d\Gamma d\Omega_Q p^\nu \frac{\partial(f - \bar{f})}{\partial\xi} + 2\int d\Gamma d\Omega_Q S = 0 \tag{26}$$

where

$$T^{\mu\nu}_f = \mathrm{diag}(E^2/2, E^2/2, E^2/2, -E^2/2) \tag{27}$$

is the energy momentum tensor for the field. We may solve for the electric field by employing the same procedure as in [15], and by employing the symmetry $\bar{f}(\tau, \xi, p_T, \theta) = f(\tau, -\xi, p_T, \theta)$. We thus obtain the equation governing the decay of the source field to be

$$\frac{dE(\tau)}{d\tau} - \frac{2g\gamma}{2\pi} \int_0^\infty dp_t p_t^2 \int_0^\infty d\xi \sinh\xi \int_0^{\pi/2} d\theta \sin\theta [f - \bar{f}] + \frac{4\pi}{7} \bar{a} |E(\tau)|^{3/2} = 0 \tag{28}$$

where $\bar{a} = a\zeta(5/2)\exp(0.25/\alpha)$, $a = c(g/2)^{5/2} \frac{\gamma}{(2\pi)^3}$ and $c = \frac{1}{(4\pi)^3)}$. Finally, $\zeta(5/2) = 1.342$ is the Reimann zeta function.

Equations (37) and (26) are as yet underdetermined since the local temperature $T(\tau)$ is free. In order to fix the form of $T(\tau)$, we appeal to the relaxation time approach that we are employing, and assume that the the particle energy density differs negligibly from the equilibrium energy density. We may then relate, by an ansatz, the proper energy density which is defined by

$$\epsilon(\tau) = \int d\Gamma d\Omega_Q (p^\mu u_\mu)^2 (f + \bar{f}) \tag{29}$$

to the temperature by its equilibrium value, whence,

$$T(\tau) = (\frac{15}{(7\gamma\pi^3)}\epsilon(\tau))^{1/4}. \tag{30}$$

It may be mentioned that the weaker condition of energy-momentum conservation that we have employed here , in fact, satisfies the Yang-Mills equations $D_\mu \vec{F}^{\mu,\nu} = \vec{j}^\nu$ as well.

Finally, we pause briefly to discuss the effects of hard thermal loops that have been emphasized recently [24]. They have been derived from the classical transport equation as well by Kelly *et al* [25,26]. The latter derivation, which is important for us here, is based on an analysis of the Vlasov equation which properly describes the expansion of an already equilibrated gas, in a back-ground field but with with no source or sink. It is, therefore, necessary to determine whether the more complete transport equation such as the one that we have here will have any effect on the results of Ref. [25,26]. Conversely, it is also necessary to determine how the hard thermal loops will affect the purely classical results that will be obtained here. Let us recall that the derivation of Kelly et al [25,26] consists of a systematic expansion of the distribution function as well as the background field term in powers of the coupling constant. The zeroth order term is merely the Fermi-Dirac term for the quarks. Since we are solving here the approach to the equlibrium, there is necessarily a dependence of the distribution functions and other physical quantities on g. It remains to disentangle the contribution coming from the hard thermal loops. Indeed, observe that the two extra terms that we have at hand here are of higher order in g. First of all, the source term is non-analytic and does not even admit an expansion in powers of g [27]. The other collision

term is easily seen to be of order g^4 or higher. We thus conclude that the conclusions of Ref. [25,26] remain unaffected, and we may take over those results *in toto*, supplementing our classical results.

III. COMPUTATIONAL PROCEDURE

We have adopted here a double-self consistent method to determine $(f, \epsilon, ...)$,following the work of Banerjee et al [15] in the Abelian case. The procedure follows the scheme $\{T(\tau)_{trial},\, E(\tau)_{trial}\} \;\rightarrow \{f, \bar{f}\; E(\tau)\} \;\rightarrow \{f, \bar{f}\} \;\rightarrow T(\tau) \;\rightarrow ...$ by repeated use of equations (20), (28), (20), (30). The iteration terminates as soon as a convergence is established in the solutions for $E(\tau), T(\tau)$. All the desired quantities are thereby consistently determined.

IV. RESULTS AND DISCUSSIONS

Before we proceed to present and discuss the results, a few comments about the choice of the value of the parameters in the computation. We put $g = 4$ throughout our calculations. Since lattice computation results predict a phase transition from the baryonic phase to the quark-gluon phase at densities $\sim 5 - 10\, GeV$, it has also been customary to take an initial energy density in the same range in the Flux tube model. However, greater care needs to be taken before the initial energy densities in URHIC are chosen. Indeed, a fairly reliable estimate of the time required for the partons to be produced in central collisions after the two nuclei have suffered the maximum overlap is $\sim 0.05\ - 0.1\, fm$; a simple dimensional analysis leads to a value of the initial energy density to be $(1/2)E_0^2 \approx 500\ - 1000\, GeV/fm^3$, the precise value depending on the the magnitude of g in $\sqrt{gE} = \frac{1}{\tau_0}$. Since in the colour plate model, we set the zero of time not at maximum overlap, but at the instant when the plates have completely crossed each other, we shall take the initial energy density $\epsilon = 300 GeV/fm^3$. We shall also be guided by the results of Geiger and Kapusta [17] in our choice of the values of τ_c, and take $\tau_c = 0.2 fm$, to be a realistic value [28]. The scale for the hydrodynamic limit will be set by the choice for τ_c as well as the formation time τ_0. A typical value for the

analysis at hand is 0.001 fm, which we employ here. Finally, we study the other extreme case, the collisionless limit, by the choice $\tau_c = 5fm$. It is instructive to compare how the realistic regime behaves in relation to the results that we obtain for the two extreme limits.

We have studied the decay of the chromoelectric field, the evolution of the particle energy and number densities, the evolution of the distribution function and its approach to the equlibrium state. We have compared them with the corresponding Abelian results. In addition, we have also evaluated a quintessentially non-Abelian quantity, *viz*, the expectation value of the angle that the quark charge makes with the direction of the chromoelectric field. In the current model the chromoelectric field not only decays to produce the $q\bar{q}$ pairs, but is itself built up by the receding nuclei which act as colour plates. We have, therefore, also studied the particle energy per unit transverse area as a function of ordinary time to highlight this feature.

A. Discussion of the results

We shall present the results of our analysis in the three regimes corresponding to the hydrodynamic, the realistic, and the collisionless cases. Comparison between the Abelian and the non-Abelian systems will be taken up subsequently.

Consider the hydrodynamic limit first. The physical quantity of utmost importance is the chromoelectric field, whose dynamics is most readily determined in the Flux tube model at hand. Indeed, apart from determining the production and the acceleration of the partons in the Flux tube model at hand, it has an additional role which has been emphasized by Svetitsky [29]: the charms which are produced in the pre-equilibrium stage would not only interact with the gas of light quarks and gluons, before either forming a J/ψ or an open charm mesonic state, but will also be influenced by the background field. Thus the study the evolution of the mean electric field in the pre-equilibrium stage acquires an added importance. It is a merit of the Flux tube model that we can readily determine the evolution of the background electric field. Note that in contrast, no such information can be extracted

in the more microscopic models such as that of Geiger and Kapusta [17]. In fig. 1 the decay of the field is shown. Recall that $\tau_c = .001fm$. As can be seen from Fig. 1, the electric field has hardly decayed at all, with a percentage decay less than 2 %, even at $\tau = 1.5fm$. The corresponding particle energy density, shown in Fig. 4 is also negligibly small, with the ratio with respect to the initial energy density being $\sim 10^{-5}$. The Abelian situation is an order of magnitude better, but is still hopelessly small. And indeed, the number density is also very small, and as shown in Fig. 7 gets saturated at $\sim .05/fm^3$. and the temperature also stabilizes to a value $\sim 40MeV$, which is much less than the temperatures required. Clearly, it is very unlikely that the plasma does not go through a prequilibrium phase.

The behaviour of the system for $\tau_c = 0.2$ and $5fm$ is in sharp contrast to the case discussed above. There is a significant decay of the electric field, which is shown in Figs. 2 and 3 for the two respective values of τ_c. The electric field decays by about 15% of the original value, at $\tau = 1.5fm$. The corresponding energy densities are also not very different, and yield $\sim 10{-}15\%$ of the initial field energy density (see Figs. 5 and 6). The corresponding temperatures are $\sim 300MeV$, which is quite realistic. Note that the parton cascade model [17] predicts a value about 300 MeV at $2.5fm$. The real difference between the collisionless case and the "realistic" case is in the number density. Indeed as Figs. 8 and 9 show, the plasma produced for $\tau_c = 0.2fm$ has a number density $\sim 15 - 20/fm^3$, the corresponding number for $\tau_c = 5fm$ is three to four times smaller. In other words, the system is more dense in the former case than the latter. Clearly, this distinction should show up in signatures such as dilepton production which are sensitive to both the number as well as the energy density.

It thus follows that the flux tube model, with the incorpration of the colour degrees of freedom, definitely rules out instantaneous equilibration as envisaged in the approach of Baym [9]; further, it also distinguishes the collisionless limit from realistic values of equilibration time.

Finally, it should be emphasized that the flux tube model as employed here not only pumps in the field energy to particle production, but also contributes to the field energy

by virtue of the recession of the colour charged nuclei from each other. In fact, a crude estimate of the time required to convert all the plate energy to the field energy turns out to be $\sim 5fm$ for $200GeV$/nucleon. In any case, it is therefore misleading to interpret the ratios we have shown in Figs. 4-6 as the fraction of the total field energy that has gone into the particles. To emphasize this, we have evaluated the dependence of the field and particle energies per unit area by integrating over the contribution along the longitudinal direction. The results are shown in Fig. 10, from which it is clear that there is a lot more energy in the field than in the particles. For the same reason, it is also misleading to interpret the field energy density at $\tau = 0$ as the counterpart of the energy densities employed in lattice computations to study the transition from the hadronc to the QGP phase.

B. Comparison with Abelian results

It is clear from Figs. 1-9 that the Abelian and the non-Abelian results bear little resemblence, belying the expectation that the "Abelian dominance" holds in this case. The difference is most dramatically highlighted at $\tau_c = 0.2fm$. Whereas almost all the initial field has decayed in the Abelian case, only 30% has done so in the non-Abelian case. Accordingly, The particle energy density is larger by a factor of ~ 5 , the number density by a factor of ~ 4, and yields an abnormally large value of$\sim 800MeV$ for the temperature. Earlier Abelian calculations [12,15,16] yielded reasonable values for the temperature because of unrealistic values for the initial field energy. It is a universal feature that the non-Abelian plasma is rarer and cooler than its Abelian counterpart. There are other significant features. Consider the hydrodynamic case. It is seen that although the decay in the non-Abelian field is more than the Abelian field, the corresponding energy density is smaller than in Abelian case. Indeed, in the hydrodynamic limit, the Abelian ananlysis yields a temperature $\sim 200MeV$, in contrast to $\sim 40MeV$ in the non-Abelian case. Thus the incorporation of the colour degree of freedom rules out instantaneous hydrodynamic evolution. Of course, a colourless plasma does not have reasonable temperature for any other τ_c.

Yet another interesting aspect that emerges from our studies is in the close interplay between the value of τ_c and the colour degree of freedom. While the field decays faster in the non-Abelian case in the hydrodynamic limit, the trend reverses at $\tau_c = 0.2fm$ and gets restored in the collisionless limit. In contrast, the particle energy density is smaller for a coloured plasma in hydrodynamic limit. Further, it continues to be so at $\tau_c = 0.2fm$ but becomes larger than the Abelian case in the collisionless limit. These non-trivial manifestations of the colour charge may be expected to have an important bearing on the other bulk properties of the plasma.

In order to gain some insight into the above features, we have also evaluated the expectation value of $\Theta \equiv < cos^2\theta >$ which yields the rms value that the particle charge makes with the field direction in the colour space. Note that the above quantity is gauge invariant, and hence physical. Fig. 11 shows $\Theta(\tau)$ for $\tau_c = 0.2fm$. It may be seen that the value saturates around 0.25, corresponding to $\theta \sim \pi/3$. The corresponding value in the non-Abelian case is strictly zero. The effect of the background field is thus reduced, leading to dominance of the collision term in the expansion of the particles. Indeed, it shows up most clearly in the approach to the equilibrium, where one now expects that the plasma equilibrates the fastest in the direction normal to that of the field (in the colour space). This is corroborated as may be seen in Fig. 12.

V. CONCLUSION

To conclude, we have studied the production and the equilibration of a genuinely non-Abelian plasma with the colour degree of freedom incorporated in both the source and the background field term in the transport equation. In the $SU(2)$ gauge theory that we have considered here, the distribution function is defined in the extended phase space. We find that this approach recaptures in an elegant manner many of the findings of the more microscopic parton cascade model. It has the further advantage that it indeed exhibits the colour degree of freedom manifestly, and allows us to compute various gauge invariant quantities.

Significantly, we also find that the Abelian approximation, employed hitherto in most studies of equilibration of QGP is rather too drastic to be used for any quantitative analysis and comparison with the experiments. The study also almost rules out instantaneous equilibration, and also strongly suggests that the collisionless limit may also not be the favoured in URHIC.

To be sure, we have not made any comparison with the experimental findings here, for the simple reason that we are as yet dealing with a simpler gauge group , and ignored the gluonic component altogether. The indications from the present study are unmistakable, though. Indeed, the particle production is enhanced because of increase in the phase space available, and for the same reason the plasma will be cooler than the Abelian counterpart. The energy goes to the colour degree of freedom, and does not simply heat the system as it would happen in a colourless plasma. If we consider the realistic $SU(3)$ case, this feature will get further accentuated; For the same initial configuration, we may expect a rarer and a cooler plasma. Of course, there are other features which are intrinsic to $SU(3)$: there is yet another Casimir invariant, and the gluon term will also have to be incorporated. These are under study at the present and will be reported else where, with a full discussion of the signatures of the QGP in the flux tube model.

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Figure captions

FIG. 1. Decay of the chromoelectric field as a function of proper time (in units of fermi), for $\tau_c = .001 fm$. The solid line refers to the non-Abelian case, and the broken line to the Abelian case.

FIG. 2. Decay of the chromoelectric field as a function of proper time (in units of fermi), for $\tau_c = .2 fm$. The solid line refers to the non-Abelian case, and the broken line to the Abelian case.

FIG. 3. Decay of the chromoelectric field as a function of proper time (in units of fermi), for $\tau_c = 5 fm$. The solid line refers to the non-Abelian case, and the broken line to the Abelian case.

FIG. 4. The particle energy density scaled w.r.t the initial field energy density as a function of proper time (in units of fermi), for $\tau_c = .001 fm$. The solid line refers to the non-Abelian case, and the broken line to the Abelian case.

FIG. 5. The particle energy density scaled w.r.t the initial field energy density as a function of proper time (in units of fermi), for $\tau_c = .2 fm$. The solid line refers to the non-Abelian case, and the broken line to the Abelian case.

FIG. 6. The particle energy density scaled w.r.t the initial field energy density as a function of proper time (in units of fermi), for $\tau_c = 5 fm$. The solid line refers to the non-Abelian case, and the broken line to the Abelian case.

FIG. 7. The particle number density as a function of proper time (in units of fermi), for $\tau_c = .001 fm$. The solid line refers to the non-Abelian case, and the broken line to the Abelian case.

FIG. 8. The particle number density as a function of proper time (in units of fermi), for $\tau_c = .2 fm$. The solid line refers to the non-Abelian case, and the broken line to the Abelian case.

FIG. 9. The particle number density as a function of proper time (in units of fermi), for $\tau_c = 5 fm$. The solid line refers to the non-Abelian case, and the broken line to the Abelian

case.

FIG. 10. The particle energy/unit transverse area (solid line) and the field energy/unit transverse area (dashed line) as a function of time (in fermi). The energy densities are in GeV/fm^2 and $\tau_c = .2fm$.

FIG. 11. $< cos^2\theta >$ as a function of proper time (in fermi) at $\tau_c = .2fm$.

FIG. 12. f/f_{eq} as a function of proper time at $p_t = 200MeV$, $\xi = 0$ and $\tau_c = 0.2fm$ for three different angles corresponding to $cos\theta = 0$ (solid line), $cos\theta = .25$ (dash line just below the solid line), and $cos\theta = 1$ (the other dash line). Note that the equlibration is fastest at $\theta = \pi/2$.